## Chapter 5.2 Algebra Law used for Query Project Improvement

This chapter will show some Algebra Laws and these laws are used to convert one Expression Tree to another equal Expression Tree, and the latter may has the more effective Physics Query Plan.

*The result of applying these Algebra Expressions is the Logical Query Plan, it is the output of Query Rewrite Phase.*

### Chapter 5.2.1 Commutation Law and Association Law

***Definition:***

1. Commutation Law: The results are always the same even there have some sequential changes of the parameters.
2. Association Law: The calculation can start from the left, also it can start from the right.

***Laws:***

Multi - Operators of Relation Algebra satisfy the Commutation Law and Association Law.

* R \* S = S \* R; (R \* S) \* T = R \* (S \* T)
* R join S = S join R; (R join S) join T = R join (S join T)
* R union S = S union R; (R union S) union T = R union (S union T)
* R intersection S = S intersection R; (R intersection S) intersection T = R intersection (S intersection T)

*(Attention: These laws are established for Set and Package.)*

***Example:***

Verify the Commutation Law: R join S = S join R:

* Assume that the tuple t exists in the result of R join S, which is to say tuple t exists in the left expression. Then tuple r must exists in Relation R, and the tuple s exists in Relation S, they must be have the same value on the common property t. Therefore when we calculate the right expression S join R, then tuple s and r will combine as the tuple t.
* Because our Relation Algebra is a package, but not set, so we must verify that if tuple t appears in the left for n times, then t should also appears in the right for n times.
* Assume that tuple t appears in the left for n times ,then tuple r in Relation R must appears for nr times, while tuple s in Relation S must appears for ns times, nr \* ns = n.
* When we calculate the right expression S join R, tupe s should appear ns times, tuple r should appear nr times, then we can get nr\*ns times t copies, then n tuple t.

***Supplement:***

Theta Join is changeable. R join c S = S join c R, as long as the condition is meaningful, then Theta Join also satisfies the Association Law.

***Example:***

Assume that we have three Relation R(a, b), S(b, c), T(c, d), the expression:

[ R join (R.b > S.b) S ] join (a < d) T

Here we can not calculate Relation S join T first, since attribute a and d do not belong to Relation S and Relation T. So when we use the Theta Join, we need to pay attention to it.

### Chapter 5.2.2 Law Selection

***Principle:***

Since the Selection Operation can be used to decrease the size of Relation, so the most important rule to process the effective query is that as long as we do not change the result of expression, then we can move the Selection Operation down as lower as we can.

*(Push Down Selection is the main method to operate Query Optimizer.)*

***Law:***

The first two laws that relates with Selection Operator is the Decomposition Operation.

* Selection (c1 and c2) (R) = Selection c1 (Selection c2 (R))
* Selection (c1 or c2) (R) = (Selection c1 (R)) union (Selection c2 (R)) *(R is the package, since if R is set, then the duplicates will not be removed.)*
* Selection c1 (Selection c2 (R)) = Selection c2 (Selection c1 (R)) *(The Sequence of c1 and c2 is flexible, normally we can exchange the Sequence of c1 and c2.)*

***Example:***

R(a, b, c) is a Relation. Then

* Selection (a = 1 OR a = 3) AND (b < c) (R) =>
* Selection (a = 1 OR a = 3) [ Selection (b < c) (R) ] =>
* Selection (a = 1) [ Selection (b < c) (R) ] Union Selection (a = 3) [ Selection (b < c) ]

*(For the division of OR operator, it requires that the parameter is set and use Union.)*

*The Other Way Around is:*

Selection (b < c) [ Selection (a = 1 OR a = 3) (R) ] =>

Selection (b < c) { [ Selection (a = 1) (R) ] Union [ Selection (a = 3) (R) ] }

***Instruction:***

Other law related with Selection permits us to proceed Push Down Selection for Unary Operator: Product, Union, Intersection, Difference, Join. There have three types law, which can be decided by whether each parameter can be chosen or a must:

1. Union, Selection Operator must be pushed down to two parameters.
2. Difference, Selection Operator must be pushed down to the first parameter, and for the second parameter can be chosen.
3. For other Operator, only require that Selection Operator be pushed down to one of the parameters.
4. For Join and Product, there is no meaning to pushed down Selection Operator to two parameters, since the parameter can have or not have the selected attribute.

***Law:***

* Selection c (R Union S) = Selection c (R) Union Selection c (S) => Union
* Selection c (R - S) = Selection c (R) - (S) => Difference
* Selection c (R - S) = Selection c (R) - Selection c (S) => Difference

***Law:***

The laws below permits to push down Selection to one or two parameters. For Selection c, we just push down the Selection to the Relation includes all attributes. Assume that the Relation R includes all attributes mentioned in Condition c, then:

* Selection c (R \* S) = Selection c (R) \* S => Product
* Selection c (R Join S) = Selection c (R) Join S => Union
* Selection c (R Join [d] S) = Selection c (R) Join [d] S => Union
* Selection c (R Intersection S) = Selection c (R) Intersection S => Intersection

If Condition c only relates with the attribute in S, then:

* Selection c (R \* S) = R \* Selection c (S) => Product

***Law:***

If Relation R and S includes the attribute in Condition c, then we can use the law below:

* Selection c (R Join S) = Selection c (R) Join Selection c (S)

*Attention:*

If the Operator is Product or Join (D), we can not use this kind of Law since Relation R and S has no common attributes. But for Intersection, this kind of law is always useful, since the mode in Relation R and S are the same.

***Example:***

Consider the Relation R(a, b), S(b, c) and expression

*Selection (a = 1 OR a = 3) AND (b < c) (R Join S)*, here condition b < c can only be used on Relation S, but the condition a = 1 and a = 3 can only be used on Relation R. Therefore:

*Selection (a = 1 OR a = 3) [ Selection (b < c) (R Join S) ]*

After that, we can push OR condition downwards, and we get:

*Selection (a = 1 OR a = 3) [R Join Selection (b < c) (S)]*

At last, we get:

*Selection (a = 1 OR a = 3) (R) Join Selection (b < c) (S)*

***Normal Law:***

1. Selection Operation for random Empty Relation is empty.
2. For the condition C always equals to true, then Selection c (R) = R.
3. If Relation R is empty, then R Union S = S.

### Chapter 5.2.3 Push Down Selection

### Chapter 5.2.4 Projection Law

### Chapter 5.2.5 Join and Product Law

### Chapter 5.2.6 Eliminate Duplication Law

### Chapter 5.2.7 Grouping and Aggregation Law